ECE 2400 Computer Systems Programming
Fall 2021

Topic 2: C Recursion

School of Electrical and Computer Engineering
Cornell University

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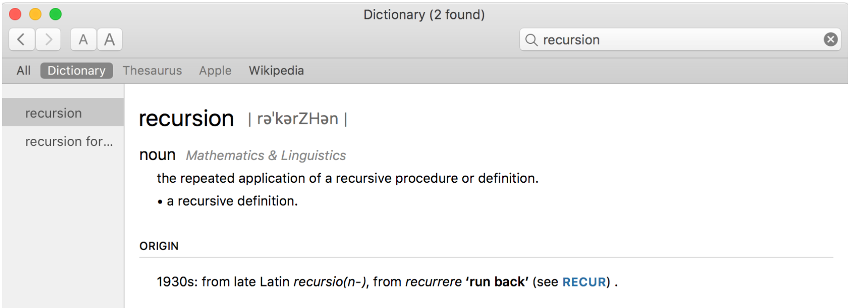
Our goal is to understand what the word “recursion” means, so let’s look up “recursion” in the dictionary ...



- Recursion is when the algorithm is defined in terms of itself
- No new syntax or semantics
- Understanding recursion simply involves applying what we have already learned with respect to functions, conditionals, iteration

1. Single Recursion

Recall from mathematics, the factorial of a number (n!) is:

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \times (n-1)! & \text{if } n > 0 \end{cases}$$

So in other words:

$0!$	$=$		$=$	1
$1!$	$=$		$=$	1
$2!$	$=$	1×2	$=$	2
$3!$	$=$	$1 \times 2 \times 3$	$=$	6
$4!$	$=$	$1 \times 2 \times 3 \times 4$	$=$	24
$5!$	$=$	$1 \times 2 \times 3 \times 4 \times 5$	$=$	120

We can write a function to calculate factorial using a `for` loop:

```c
int factorial( int n ) {
  int result = 1;
  for ( int i = 1; i <= n; i++ )
    result = result * i;
  return result;
}
```

- The loop implementation does not really resemble the original mathematical formulation
- The mathematical formulation is inherently recursive
- Can we implement factorial more directly using recursion?

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \times (n-1)! & \text{if } n > 0 \end{cases}$$

We can use the exact same "by-hand" execution approach we learned in the previous topic to understand recursion.

```
int factorial( int n )
{
  // base case
  if ( n == 0 ) {
    return 1;
  }
  // recursive case
  if ( n > 0 ) {
    return n *
      factorial(n-1);
  }
}

int main()
{
  int a = factorial(3);
  return 0;
}
```

stack

Questions:

- What if n is negative?
- What if the execution arrow reaches end of a non-void function without encountering a return statement?

2. Multiple Recursion

Recall from mathematics, the Fibonacci sequence is a sequence of integers such that every number after the first two is the sum of the two preceding ones:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...

The numbers in the Fibonacci sequence are called "Fibonacci numbers". By definition, the first two numbers in the Fibonacci sequence are 0 and 1. Ancient scholars realized the importance of this sequence in both mathematics and nature. Fibonacci sequences can be found in the arrangement of leaves on a stem or patterns in a pine cone.

We can write a function to calculate the n^{th} Fibonacci number using a `for` loop:

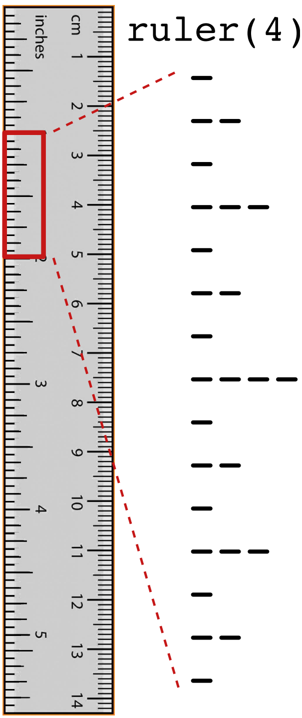
```c
int fib( int n ) {

  // by definition
  if (n == 0) return 0;
  if (n == 1) return 1;

  int fib_minus2 = 0;
  int fib_minus1 = 1;
  int result     = 0;

  for ( int i=2; i<=n; i++ ) {

    result = fib_minus1
           + fib_minus2;

    fib_minus2 = fib_minus1;
    fib_minus1 = result;

  }
  return result;
}
```

Can we implement factorial more elegantly using recursion?

Illustrating call tree for `fib`

3. Writing a Recursive Function

Write pseudo-code for a recursive function which draws the tick marks on a vertical ruler. The middle tick mark should be the longest and mark the 1/2 way point, slightly shorter tick marks should mark the 1/4 way points, even slightly shorter tick marks should mark the 1/8 way points and so on. The function should take one argument: the height of the middle tick mark (i.e., the number of dashes). The function should always return 0.




```
int ruler( int height ) {
```

```
ruler(1)
-
```

```
ruler(2)
-
--
-
```

```
ruler(3)
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```

```
ruler(4)
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```

```
ruler(5)
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```

- Step 1: Work an example yourself
  - height = 2, height = 3
- Step 2: Write down what you just did
  - What is the base case?
  - What is the recursive case?
- Step 3: Generalize your steps
  - for any height
- Step 4: Test your algorithm
  - does it work for height = 4?
- Step 5: Translate to pseudocode

Think about the recursive call tree?

Manually work through example ruler